Newton's Law of Cooling: we'll do one in group. The one we started last week is posted on website.

ex)

we put a 80°F cup of coffee in a room that's 60°F. It cools to 75° F in 1 min. What is its temp after 10 mins?

$y'(t) = K(m - y(t))$ $\qquad m = 60$

so $\quad y(t) = m + Ae^{-Kt}$

$y(t) = 60 + Ae^{-Kt}$ $\qquad \boxed{y(0) = 80}$

$80 = y(0) = 60 + Ae^{-K\cdot 0} = 60 + A$

$A = 20$ $\qquad y(t) = 60 + 20e^{-Kt}$

$\boxed{y(1) = 75}$

$75 = y(1) = 60 + 20e^{-K\cdot 1} = 60 + 20e^{-K}$

$\frac{15}{20} = \frac{20e^{-K}}{20}$

$\frac{3}{4} = e^{-K}$

$\ln(3/4) = -K$ $\qquad K = -\ln\left(\frac{3}{4}\right)$

$y(t) = 60 + 20e^{\ln(3/4)t} = 60 + 20\left(\frac{3}{4}\right)^t$

$y(10) = 60 + 20 \cdot \left(\frac{3}{4}\right)^{10} \approx$

$= 60 + 20 \cdot (.056)$

$60 + 1.1262\ldots$

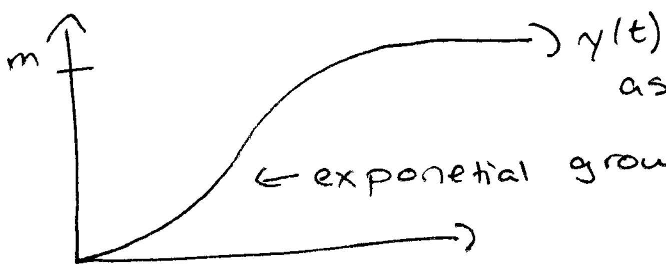
$\approx \boxed{61.126^\circ}$ F

logistic equation

$$y'(t) = Ky(t)\left(1 - \frac{y(t)}{m}\right)$$

$$y(t) = \frac{m}{Ae^{-Kt}+1}$$

m: limiting value



as $t \to \infty$, $y(t) \to m$

← exponetial growth for a bit, then levels out

ex) $y'(t) = 4y(t)\left(1 - \frac{y(t)}{10}\right)$, $y(0)=5$, find $y(\ln 2)$

$m=10$
$K=4$

$$\boxed{y(t) = \frac{10}{Ae^{-4t}+1}}$$ gen solution

~~say $y(0)=5$~~

$$y(0)=5 = \frac{10}{Ae^{-4\cdot 0}+1} = \frac{10}{A+1}$$

$5(A+1) = 10$

$A+1 = 2$

$A=1$

so $\boxed{y(t) = \frac{10}{e^{-4t}+1}}$

particular sol'n

$$y(t) = \frac{10}{e^{-4t}+1}$$

$$y(\ln(2)) = \frac{10}{e^{-4\ln(2)}+1} = \frac{10}{e^{\ln(2)\cdot(-4)}+1}$$

$$= \frac{10}{2^{-4}+1}$$

$$= \frac{10}{\frac{1}{2^4}+1} = \frac{10}{\frac{1}{16}+1}$$

$$= \frac{10}{\left(\frac{17}{16}\right)}$$

$$= 10 \cdot \frac{16}{17}$$

$$= \boxed{\frac{160}{17}}$$

ex) Bacteria is growing in a closed area with maximum capacity 200 mg, at a rate given by the logic equation. initially, there is 10 mg. after 1 day, this increases to 50 mg. ~~how long does it take to reach the max capacity?~~ ~~what~~ how much bact is there after 2 days?

$$y'(t) = K \cdot y(t)\left(1 - \frac{y(t)}{m}\right)$$

$$y(t) = \frac{m}{Ae^{-Kt}+1} \qquad m = 200$$

$$y(t) = \frac{200}{Ae^{-Kt}+1}$$

$$y(0) = 10 = \frac{200}{Ae^{-K\cdot 0}+1} = \frac{200}{A+1}$$

$$10(A+1) = 200$$

$$1 + A = 20$$

$$\boxed{A = 19}$$

$$y(t) = \frac{200}{19e^{-kt}+1}$$

$$y(1) = 50$$

$$50 = \frac{200}{19e^{-k}+1}$$

$$\frac{50}{50}\left(19e^{-k}+1\right) = \frac{200}{50}$$

$$19e^{-k}+1 = 4$$

$$19e^{-k} = 3$$

$$e^{-k} = \frac{3}{19}$$

$$-k = \ln\left(\frac{3}{19}\right)$$

$$\boxed{k = -\ln\left(\frac{3}{19}\right)}$$

$$y(t) = \frac{200}{19e^{\ln(3/19)t}+1} = \frac{200}{19\left(\frac{3}{19}\right)^t+1} = y(t)$$

$$y(2) = \frac{200}{19\cdot\left(\frac{9}{361}\right)+1} = \frac{200}{\frac{9}{19}+1}$$

$$= \frac{200}{\left(\frac{28}{19}\right)}$$

$$= 200\cdot\frac{19}{28} =$$

$$100\cdot\frac{19}{14} = \frac{1900}{14}$$

$$= \boxed{\frac{950}{7}}$$

$$\approx \boxed{135.7\,mg}$$